ACMS Applied Math Seminar

Ruriko Yoshida

Thursday, October 15

154 Hurley Hall

3:30- 4:30 PM

Extremal Positive Semidefinite Matrices for Graphs Without K_5 Minors

For a graph G with p vertices the closed convex cone $\mathbb{S}^p_{\succeq0}(G)$ consists of all real positive semidefinite $p\times p$ matrices with zeros in the off-diagonal entries corresponding to nonedges of ~G. The extremal rays of this cone and their associated ranks have applications to matrix completion problems, maximum likelihood estimation in Gaussian graphical models in statistics, and Gauss elimination for sparse matrices.

In this talk, we will show that the normal vectors to the facets of the (±1)-cut polytope of G specify the off-diagonal entries of extremal matrices in $\mathbb{S}^p_{\succeq0}(G)$ for a weakly bipartite graph G.

We will also show that the constant term of the linear equation of each facet-supporting hyperplane is the rank of its corresponding extremal matrix in $\mathbb{S}^p_{\succeq0}(G)$. Furthermore, we show that if G is series-parallel then this gives a complete characterization of all possible extremal ranks of $\mathbb{S}^p_{\succeq0}(G)$, consequently solving the sparsity order problem for series-parallel graphs.
This is joint work with Caroline Uhler at IST/MIT and Liam Solus at University of Kentucky (IST/MIT).