An Exact Gibbs Sampler of Stochastic Differential Equations

Continuous-time diffusions (stochastic differential equations) are popular models in fields like finance, biology and chemistry. These serve as priors over hidden trajectories, and given noisy observations, define posterior distributions over latent paths and process parameters. Inference is complicated by the intractable transition probabilities, and one typically has to use approximate MCMC schemes that discretize time. Beskos and Roberts 2005, developed a rejection-sampling algorithm with no such approximation error. While exact, large observation intervals and informative measurements can lead to high rejection rates and slow mixing. We develop a Gibbs sampler that ameliorates some of these issues. At a high level, our algorithm can be understood as alternately sample a new discretization of time given a diffusion path, and then a new path given the discretization. We show the first step is easy using Poisson thinning, and the second involves standard discrete-time techniques.