Department of Applied and Computational Mathematics and Statistics Colloquium

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The Mathematics of IDEAL

Iterative Decomposition of water and fat with Echo Asymmetry and Least-squares estimation (IDEAL) is a technique proposed by Reeder et al in 2004 to measure water and fat concentrations in tissue using Magnetic Resonance (MR). For gradient echo sequences, the MR signal is modeled as the superposition of the emissions of a single-frequency water spectrum and the multi-frequency fat spectrum modulated by a factor that accounts for the field inhomogeneity and T2* decay.

In practice, estimating the concentrations from the MR signal is challenging. Since both the field inhomogeneity and T2* decay are unknown the problem is highly non-linear. Several methods have been proposed to estimate these parameters and recover the concentrations from the MR signal. They typically promote smoothness on the field inhomogeneity using graph cut methods and multiscale decompositions. Although they perform well, in some critical cases they incorrectly estimate the concentrations, and in general the estimates of the field inhomogeneity and T2* decay are much less accurate than those for the concentrations. This is critical in other applications, such as evaluating iron overload or for quantitative susceptibility mapping. In addition, the relation between the results obtained by these methods and the exact solution characterized by the mathematical model has not been systematically studied.

In this talk, I will present a mathematical analysis of IDEAL that addresses the problem of exact recovery of the concentrations and parameters and enables a new approach to fit the IDEAL model. In particular, under suitable conditions the concentrations may be recovered exactly even when the field inhomogeneity and T2* decay are not. The recovery procedure we propose leads to a non-convex optimization problem. We study the effect of the initial point to the optimization algorithm and its robustness to noise. Finally, I discuss how our work has the potential to extend IDEAL beyond measuring water and fat. Experiments in silico and in vivo support our theoretical results.

Joint work with S. Uribe (Department of Radiology, School of Medicine, UC) and C. Arrieta (Biomedical Imaging Center, UC).

Monday, February 18, 2019
4:15 PM – 5:15 PM
127 Hayes-Healy Center

Colloquium Tea 3:45 PM to 4:15 PM 101A Crowley Commons Room