Extensions of Real Atomic Gauges for Complex Signal Recovery

Solving discrete linear inverse problems is one of the cornerstones of modern science and engineering. In abstract terms, these problems seek to recover an unknown vector from an incomplete set of linear measurements. When the object is a sparse convex combination of a known collection of atoms, the gauge associated to the convex hull of this collection, i.e., the atomic gauge, can be minimized subject to data consistency constraints to recover the original vector. In some practical applications, such as magnetic resonance imaging, the vector is complex-valued and it is the magnitude vector, i.e., the vector containing the magnitude of the components, that is a sparse convex combination of known real-valued atoms.

In this talk, I will present a method to extend the collection of real-valued atoms by considering their modulations by a collection of suitable phases. The resulting atomic gauge can be then applied to complex vector recovery. Furthermore, under minor assumptions, both the resulting atomic gauge and its proximal map can be evaluated by solving a convex optimization problem. As a consequence, the complexity of using the gauge associated to a collection of modulated atoms is comparable to that of using a collection of real-valued atoms. The numerical results show the advantage of using this extension for complex signal recovery.