ACMS Applied Math Seminar

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127 Hayes-Healy
3:30 PM – 4:30 PM

Divergence - and Curl-Constraint Preserving Higher Order Godunov Schemes

A study of several PDE systems found in nature show that they have involution constraints. In this work, we focus on those PDE systems that have divergence-based and curl-based involution constraints. PDE systems like Maxwell's Equations and the equations of Magnetohydrodynamics (MHD) have been known to have divergence-preserving properties. The divergence of one or more vector fields that are used in these systems are preserved, or held down to zero, as the system evolves in time. In recent years, newer classes of PDEs have emerged that preserve the curl of vector fields that are used in the time evolution of the PDE. Examples of such systems include hyperelasticity, compressible multiphase flow, Schrödinger's equation and General Relativity when cast in the FO-CCZ4 formulation.

We show that there are common ingredients in the design of constraint-preserving higher order Godunov schemes for many of these systems. One of the common ingredients consists of reconstruction strategies for vector fields that preserve the desired vectorial constraint. As a result, multidimensional reconstruction strategies for multidimensional vector fields that preserve the divergence or curl constraints will be presented. Another common building block consists of the multidimensional Riemann solver in 2D and 3D which gives us an approach for enforcing multidimensional upwinding at various locations on the mesh. It is shown that these building blocks can be combined to build constraint-preserving FV-like, DG-like and PNPM-like higher order Godunov methods with dispersion and dissipation that diminishes with increasing accuracy. We also show that careful attention to reconstruction of vector fields gives us useful insights into constraint-preserving AMR.