ACMS Applied Math Seminar

Ziyao Xu
University of Notre Dame

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154 Hurley Hall
3:30 PM – 4:30 PM

A High-Order Well-Balanced Alternative Finite Difference WENO (A-WENO) Method with the Exact Conservation Property for Systems of Hyperbolic Balance Laws

In this work, we develop a high-order well-balanced alternative finite difference weighted essentially non-oscillatory (A-WENO) method with the exact conservation property and high efficiency for a class of hyperbolic balance laws whose steady states are characterized by constant equilibrium variables. In particular, the method preserves the non-hydrostatic equilibria of the shallow water equations with non-flat bottom topography and the Euler equations in gravitational fields. Our method comprises three essential gradients. First, we adopt the finite difference framework to discretize the equations, thus we approximate the value of source terms at grid points rather than their averages on cells. Then, we rewrite the source terms in flux-gradient forms at local reference equilibrium states and discretize them using the same approach as the true flux gradient to achieve the well-balanced property. Most importantly, the exact conservation property and high efficiency are achieved through the interpolation of equilibrium variables in the A-WENO framework, which is different from the more widely used finite difference framework based on the reconstruction of fluxes. Since the equilibrium variables are constants at equilibria in the equations we study, the WENO interpolation becomes trivial for the local reference equilibrium states in the flux-gradient formulation of source terms, which is the key to efficiency and preservation of conservation property.