ACMS Applied Math Seminar

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154 Hurley Hall
3:30 PM – 4:30 PM

Transformations for Piola-mapped Elements

Many finite element spaces are not preserved by the standard pullback to the reference cell. Robust implementation therefore requires studying the relation between degrees of freedom under pushforward, in order to obtain the correct bases on a generic physical triangle [Kirby, 2018]. In this work, we extend this transformation theory to vector- and tensor-valued elements mapped by the contravariant Piola transform. We apply this theory, and describe its efficient implementation in Firedrake, for the the Mardal–Tai–Winther elements discretizing H(div) for Stokes–Darcy flow, and the conforming and nonconforming Arnold–Winther elements discretising H(div; S) for the stress-displacement formulation of linear elasticity. In particular, the Arnold–Winther elements were the first to stably enforce exact symmetry of the Cauchy stress tensor; we demonstrate how they may be efficiently mapped, while the few prior implementations are either custom-made for specific numerical experiments, or require the explicit element-by-element construction of the basis. Our novel implementation of these exotic elements composes inexpensively and automatically with the rest of the Firedrake code stack. We also demonstrate the effectiveness of appropriate multigrid smoothers for this system, prove convergence of Nitsche's method for the weak enforcement of traction conditions, and provide a uniform construction of all standard reference-to-physical Piola pullback maps using the finite element exterior calculus.