

A Mathematics course for advanced high school students

A typical high school mathematics curriculum progresses from algebra and geometry to precalculus and calculus. For a number of students who start algebra in 8th grade and successfully complete the above sequence by the end of 11th grade, there is the question of what math to study in the 12th grade. The National Academy of Sciences, addressing the issue of mathematical proficiency (see Adding it Up: Helping Children to Learn Mathematics (2001), list five components. Briefly, they are conceptual understanding, procedural fluency, strategic competence (formulating, representing and solving mathematical problems), adaptive reasoning (logical thought, especially in “providing a sufficient reason for”), and productive disposition (seeing that mathematics makes sense and that I can learn it). Starting with these two pieces of information, the following curriculum was developed. The goals of the course are 1) to engage in mathematical thought by generalizing the results of single variable calculus to two variables, to study the more significant level of abstraction found in vector spaces by solving systems of linear equations and the eigen-problem, and to solve dynamical systems by employing knowledge of mathematical models, and 2) to employ multiple points of view, collaborative learning, and vertical/horizontal integration with other math and science courses/topics.

Basic Overview of the syllabus

1st trimester (50 days) focuses on 2-variable calculus, especially partial derivatives, optimization, and line integrals up to Green’s theorem.

2nd trimester (50 days) studies linear systems, vector spaces, orthogonality and eigensystems.

3rd trimester (60 days) begins with linear difference equations, proceeds to dynamical systems expressed as linear differential equations, and finally as nonlinear differential equations, with both calculus and linear algebra used extensively.

In all sections of the course the use of a computer algebra or numerical system, such as MATLAB, is woven into the study.

Each trimester emphasizes a different component of proficiency, with conceptual understanding as the driving force throughout the entire course. Procedural fluency is the focus in the calculus, adaptive reasoning in the linear algebra, and strategic competence in the dynamical systems. Group projects are an important aspect of the course, especially in the study of dynamical systems.

Such a course allows an advanced student to see both the depth and breadth of mathematics, both in its pure and in its applied aspects. It also puts the student in a good position for any further study at the collegiate level which uses mathematics.